APPLICATION OF MODIFIED MAPPING-COLLOCATION METHOD TO CRACKS EMANATING FROM A CIRCULAR HOLE IN AN ORTHOTROPIC FINITE PLATE

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A modified mapping-collocation method is applied to the analysis of cracks emanating from a circular hole in an orthotropic finite plate under uniform stress. To check the effectiveness of this procedure, we present the various results for comparison with references. Then, the stress intensity factors are presented for several plate configurations of $[0_n/90_m]_s$ laminates. The results show that the modified mapping-collocation method is effectively applicable to analyzing such cracks in an orthotropic finite plate. The results also show that the stress intensity factors depend on the material orthotropy and geometry.

Key Words: Mapping-Collocation, Stress Function, Hole, Crack, Orthotropy, Stress Intensity Factor

1. INTRODUCTION

Fracture of structural components is frequently caused by the initiation and growth of one or more cracks from stress concentrated regions. Many investigators have demonstrated that stress intensity solutions can be used to predict the fracture of structural components. Therefore, a technique for calculating the stress intensity factors for cracks placed near stress concentrated regions should be useful to designers or experimental investigators. Several investigators(Bowie, 1956 ; Newman Jr., 1971 ; Hsu, 1975 ; Shivakumar and Forman, 1980) have obtained theoretical solutions for cracks emanating from a circular hole in an isotropic plate. However, relatively few studies of the problem for anisotropic materials have been reported.

Recently, the modified mapping-collocation method was developed by Bowie and Neal(Bowie and Neal, 1970) for isotropic problems. This method combines the advantages of conformal mapping techniques with boundary collocation arguments. It was extended by Bowie and Freese(Bowie and Freese, 1972) to the analysis of anisotropic problems and applied to the central crack in plane orthotropic rectangular sheet. Gandhi(Gandhi, 1972) applied this method to the analysis of an inclined crack centrally placed in an orthotropic rectangular plate.

In this paper the modified mapping-collocation method is applied to the analysis of cracks emanating from a circular hole in an orthotropic finite plate under uniform stress. To check the effectiveness of this procedure, we presented the various results for comparison with references. Then, the stress intensity factors were presented for several plate configurations of $[0_n/90_m]_s$ laminates.

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2. BASIC EQUATIONS OF TWO-DIMENSIONAL ANISOTROPIC ELASTICITY

If we assume that the complex parameters are all different, the general form of the stress function satisfying equilibrium and compatibility equations can be expressed as(Lekhnitskii, 1968).

$$F(x, y) = 2Re[F_1(z_1) + F_2(z_2)] \tag{1}$$

where

$$z_k = x + s_k y \quad (k=1, 2) \tag{2}$$

F_1 and F_2 are analytic functions of the complex variables z_1 and z_2 respectively. The complex parameters s_1, s_2 are roots of characteristic equation and are taken the positive of the imaginary part. The characteristic equation is given by (Lekhnitskii, 1968) :

$$a_{11}s^4 - 2a_{16}s^3 + (2a_{12} + a_{66})s^2 - 2a_{26}s + a_{22} = 0 \tag{3}$$

Employing the stress functions, the stress components are

$$\begin{aligned} \sigma_x &= 2Re[s_1{}^2\phi_1'(z_1) + s_2{}^2\phi_2'(z_2)] \\ \sigma_y &= 2Re[\phi_1'(z_1) + \phi_2'(z_2)] \\ \tau_{xy} &= -2Re[s_1\phi_1'(z_1) + s_2\phi_2'(z_2)] \end{aligned} \tag{4}$$

where

$$\phi_k(z_k) = F_k'(z_k) \quad (k=1, 2) \tag{5}$$

From the strain-displacement relations, a simple integration gives the displacement components u and v :

$$\begin{aligned} u &= 2Re[p_1\phi_1(z_1) + p_2\phi_2(z_2)] \\ v &= 2Re[q_1\phi_1(z_1) + q_2\phi_2(z_2)] \end{aligned} \tag{6}$$

where p_k, $q_k (k=1, 2)$ are defined by

$$p_k = a_{11}s_k^2 + a_{12} - a_{16}s_k$$
$$q_k = (a_{12}s_k^2 + a_{22} - a_{26}s_k)/s_k \quad (k=1,\ 2) \tag{7}$$

The boundary conditions of the traction type may also be expressed as

$$\begin{aligned} f_1(s) + if_2(s) &= \pm i\int_{\xi_o}^{s}(X_n + iY_n)\,ds \\ &= (1+is_1)\phi_1(z_1) + (1+is_2)\phi_2(z_2) \\ &+ (1+i\bar{s}_1)\overline{\phi_1(z_1)} + (1+i\bar{s}_2)\overline{\phi_2(z_2)} + c \end{aligned} \tag{8}$$

where X_n and Y_n are the x and y components of forces exerted upon the edge per unit area. We take the upper signs for the external contour, and the lower for the internal. The counterclockwise direction along the external and internal contours will be considered as positive. ξ_o is the initial point and s is a variable point on each contour. The bar notation is a conjugate symbol.

The boundary conditions of the displacement type may be written as

$$h_1(s) = 2Re[p_1\phi_1(z_1) + p_2\phi_2(z_2)]$$
$$h_2(s) = 2Re[q_1\phi_1(z_1) + q_2\phi_2(z_2)] \tag{9}$$

3. THEORETICAL DEVELOPMENTS

We consider straight cracks emanating from a circular hole in an orthotropic finite plate as shown in Fig. 1. The modified mapping-collocation method will be utilized to solve this problem. The stress functions ensuring traction-free conditions on the crack surface can be derived by using analytic continuation arguments of functions.

We introduce the so-called Joukowsky transformation,

$$z = \omega(\zeta) = \frac{L}{2}\left[\zeta + \frac{1}{\zeta}\right] \tag{10}$$

The above mapping function carries the unit circle and its exterior in the ζ-plane into the crack and its exterior. The other boundaries correspond to a closed contour in the ζ-plane exterior to the unit circle with co-ordinate points

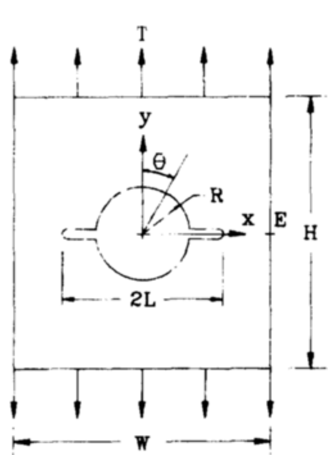

Fig. 1 Cracks emanating from a circular hole in an orthotropic finite plate under uniform stress

$$\zeta = \frac{z}{L} + \left[\left[\frac{z}{L}\right]^2 - 1\right]^{\frac{1}{2}} \tag{11}$$

We consider now the complex variables z_1, z_2, and the additional relations:

$$z_k = \omega(\zeta_k) = \frac{L}{2}\left[\zeta_k + \frac{1}{\zeta_k}\right] \quad (k=1,\ 2) \tag{12}$$

Since $z = z_1 = z_2$ on the crack, the parameter planes ζ, ζ_1, and ζ_2 coincide on the unit circle. Otherwise, ζ_1 and ζ_2 are distinct and are found from

$$\zeta_k = \frac{z_k}{L} + \left[\left[\frac{z_k}{L}\right]^2 - 1\right]^{\frac{1}{2}} \quad (k=1,\ 2) \tag{13}$$

Thus, a simple mapping function has been used and the difficult task of finding an exact mapping function carrying the physical region of Fig. 1 into a parameter region has been avoided.

For convenience, we now define the following useful notation:

$$\phi_k(z_k) = \phi_k[\omega(\zeta_k)] = \phi_k(\zeta_k),$$
$$\phi_k'(z_k) = \phi_k'(\zeta_k)/\omega'(\zeta_k) \quad (k=1,\ 2) \tag{14}$$

where

$$\omega'(\zeta_k) = \frac{L}{2}\left[1 - \frac{1}{\zeta_k^2}\right] \quad (k=1,\ 2) \tag{15}$$

Taking into account the relations in Eqs. (4) and (14), the stresses in terms of $\phi_1(\zeta_1)$ and $\phi_2(\zeta_2)$ are

$$\begin{aligned} \sigma_x &= 2Re\left[s_1^2\frac{\phi_1'(\zeta_1)}{\omega'(\zeta_1)} + s_2^2\frac{\phi_2'(\zeta_2)}{\omega'(\zeta_2)}\right] \\ \sigma_y &= 2Re\left[\frac{\phi_1'(\zeta_1)}{\omega'(\zeta_1)} + \frac{\phi_2'(\zeta_2)}{\omega'(\zeta_2)}\right] \\ \tau_{xy} &= -2Re\left[s_1\frac{\phi_1'(\zeta_1)}{\omega'(\zeta_1)} + s_2\frac{\phi_2'(\zeta_2)}{\omega'(\zeta_2)}\right] \end{aligned} \tag{16}$$

From Eqs. (6) and (14), the displacements in terms of $\phi_1(\zeta_1)$ and $\phi_2(\zeta_2)$ are

$$u = 2Re[p_1\phi_1(\zeta_1) + p_2\phi_2(\zeta_2)]$$
$$v = 2Re[q_1\phi_1(\zeta_1) + q_2\phi_2(\zeta_2)] \tag{17}$$

From Eqs. (8) and (14), the resultant—forces in terms of $\phi_1(\zeta_1)$ and $\phi_2(\zeta_2)$ are

$$\begin{aligned} f_1(s) + if_2(s) &= (1+is_1)\phi_1(\zeta_1) + (1+is_2)\phi_2(\zeta_2) \\ &+ (1+i\bar{s}_1)\overline{\phi_1(\zeta_1)} + (1+i\bar{s}_2)\overline{\phi_2(\zeta_2)} + c \end{aligned} \tag{18}$$

Let $S_{\zeta_1}^+$ and $S_{\zeta_2}^+$ denote the two parameter regions corresponding to ζ_1 and ζ_2, respectively. Their union, $S_{\zeta_1}^+ + S_{\zeta_2}^+$, will be denoted by S_ζ^+. Figure 2 shows the transformed parameter region S_ζ^+.

We introduce the following relation(Bowie and Freese, 1972):

$$\phi_2(\zeta_2) = B\overline{\phi}_1\left[\frac{1}{\zeta_2}\right] + C\phi_1(\zeta_2) \tag{19}$$

where

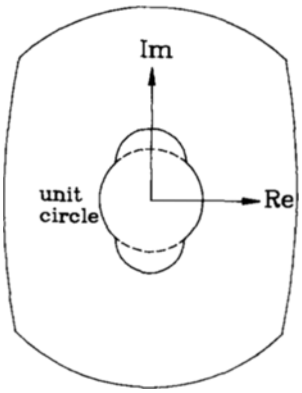

Fig. 2 ζ-transformed plane

$$\overline{\phi_1}\left[\frac{1}{\zeta}\right]=\overline{\phi_1\left[\frac{1}{\bar{\zeta}}\right]} \tag{20}$$

$$B=(\bar{s}_2-\bar{s}_1)/(s_2-\bar{s}_2),\quad C=(\bar{s}_2-s_1)/(s_2-\bar{s}_2) \tag{21}$$

Traction-free condition on the crack can be ensured if $\phi_1(\zeta)$ is analytic in the region S_ζ^+ and its inversion with respect to the unit circle(Bowie and Freese, 1972). If we assume that the total resultant-forces per unit thickness exerted on the hole boundary are zero, we can express ϕ_1 as follows(Cheong and Hong, 1987):

$$\phi_1(\zeta)=\sum_{-\infty}^{\infty}A_n\zeta^n+\sum_{n=1}^{\infty}B_n(\zeta+i)^{-n}+\sum_{n=1}^{\infty}C_n(\zeta-i)^{-n} \tag{22}$$

where A_n, B_n, and C_n are complex constants.

By considering stress symmetries and taking into account $[0_n/90_m]_s$ laminates, the stress function ϕ_1 may be rewritten as follows:

$$\phi_1(\zeta)=\sum_{-\infty}^{\infty}A_n\zeta^{2n+1}+\sum_{n=1}^{\infty}B_n\zeta(\zeta^2+1)^{-n} \tag{23}$$

where A_n and B_n are rearranged real constants. In general, $[0_n/90_m]_s$ laminates have pure imaginary complex parameters.

Finally, the problem simplifies to selecting the unknowns A_n and B_n in Eq. (23) so that the boundary conditions may be satisfied. In the analysis of an orthotropic infinite plate, A_n's are directly obtained by applying the boundary conditions at infinity(Cheong and Hong, 1987). To accomplish numerical analysis, we have to truncate the terms of Eq. (23).

Substituting Eq. (23) into Eq. (16), the stresses are

$$\sigma_x=2Re\left[\sum_{-M}^{N}S_{1n}A_n+\sum_{n=1}^{K}S_{2n}B_n\right],\ \text{etc.} \tag{24}$$

where

$$S_{1n}=\frac{s_1^2}{\omega'(\zeta_1)}(2n+1)\zeta_1^{2n}+\frac{s_2^2}{\omega'(\zeta_2)}\{C(2n+1)\zeta_2^{2n}-B(2n+1)/\zeta_2^{2n+2}\}$$
$$S_{2n}=\frac{s_1^2}{\omega'(\zeta_1)}\frac{(1-2n)\zeta_1^2+1}{(\zeta_1^2+1)^{n+1}}+\frac{s_2^2}{\omega'(\zeta_2)}\frac{B\{(2n-1)\zeta_2^{2n-2}-\zeta_2^{2n}-\zeta_2^{2n}\}+C\{(1-2n)\zeta_2^2+1\}}{(\zeta_2^2+1)^{n+1}} \tag{25}$$

Substituting Eq. (23) into (17), the displacements are

$$u=2Re\left[\sum_{-M}^{N}D_{1n}A_n+\sum_{n=1}^{K}D_{2n}B_n\right],\ \text{etc.} \tag{26}$$

where

$$D_{1n}=p_1\zeta_1^{2n+1}+p_2\{B/\zeta_2^{2n+1}+C\zeta_2^{2n+1}\}$$
$$D_{2n}=p_1\frac{\zeta_1}{(\zeta_1^2+1)^n}+p_2\left[B\frac{1}{\zeta_2(1/\zeta_2^2+1)^n}+C\frac{\zeta_2}{(\zeta_2^2+1)^n}\right] \tag{27}$$

Substituting Eq. (23) into Eq. (18), the resultant—forces are

$$f_1=\mp\int_{\theta_0}^{s}Y_n ds=2Re\left[\sum_{-M}^{N}F_{1n}A_n+\sum_{n=1}^{K}F_{2n}B_n\right]+c_1$$
$$f_2=\pm\int_{\theta_0}^{s}X_n ds=2Re\left[\sum_{-M}^{N}F_{3n}A_n+\sum_{n=1}^{K}F_{4n}B_n\right]+c_2 \tag{28}$$

where

$$F_{1n}=\zeta_1^{2n+1}+\{B/\zeta_2^{2n+1}+C\zeta_2^{2n+1}\}$$
$$F_{2n}=\frac{\zeta_1}{(\zeta_1^2+1)^n}+B\frac{1}{\zeta_2(1/\zeta_2^2+1)^n}+C\frac{\zeta_2}{(\zeta_2^2+1)^n}$$
$$F_{3n}=s_1\zeta_1^{2n+1}+s_2\{B/\zeta_2^{2n+1}+C\zeta_2^{2n+1}\}$$
$$F_{4n}=s_1\left[\frac{\zeta_1}{(\zeta_1^2+1)^n}\right]+s_2\left[B\frac{1}{\zeta_2(1/\zeta_2^2+1)^n}+C\frac{\zeta_2}{(\zeta_2^2+1)^n}\right] \tag{29}$$

Truncating the unknown terms A_n and B_n in Eq. (23) so that the boundary conditions are satisfied with sufficient accuracy, the stress functions may be determined.

4. APPLICATION OF MODIFIED MAPPING-COLLOCATION METHOD

The least square collocation procedure suggested in the literature (Bowie and Freese, 1972) was utilized. In multiply connected regions there is a difficulty associated with constants of integration in f_1+if_2. Considering a traction-free condition on the crack and circular hole, the constants of integration for the internal contour have been chosen as zero. Therefore, the constants of integration for the internal contour are as follows:

$$c_1=0,\quad c_2=0 \tag{30}$$

Taking into account stress symmetries and the resultant force acting on the segment of the real axis from the crack tip $z=L$ to the point E, the constants of integration for the external contour can be easily determined. Then, the constants of integration for the external contour are as follows:

$$c_1=-WT/2,\quad c_2=0 \tag{31}$$

We can rewrite Eq. (28) as follows:

$$2Re\left[\sum_{-M}^{N}F_{1n}A_n+\sum_{n=1}^{K}F_{2n}B_n\right]=-c_1\mp\int_{\theta_0}^{s}Y_n ds$$
$$2Re\left[\sum_{-M}^{N}F_{3n}A_n+\sum_{n=1}^{K}F_{4n}B_n\right]=-c_2\pm\int_{\theta_0}^{s}X_n ds \tag{32}$$

We take the point E and point $(R, 0)$ as the initial points for the external and internal contours respectively(see Fig. 1). At the initial points, Eq. (32) can be reduced as follows:

$$2Re\left[\sum_{-M}^{N} F_{1n}A_n + \sum_{n=1}^{K} F_{2n}B_n\right] = -c_1$$
$$2Re\left[\sum_{-M}^{N} F_{3n}A_n + \sum_{n=1}^{K} F_{4n}B_n\right] = -c_2 \quad (33)$$

If we take ξ_i as the i—th collocation point, Eq. (32) can be expressed as follows:

$$2Re\left[\sum_{-M}^{N} F_{1n}\Big|_{\xi i}^{\xi i} A_n + \sum_{n=1}^{K} F_{2n}\Big|_{\xi i}^{\xi i} B_n\right] = \mp\int_{\xi i-1}^{\xi i} Y_n ds$$
$$2Re\left[\sum_{-M}^{N} F_{3n}\Big|_{\xi i-1}^{\xi i} A_n + \sum_{n=1}^{K} F_{4n}\Big|_{\xi i-1}^{\xi i} B_n\right] = \pm\int_{\xi i-1}^{\xi i} X_n ds \quad (34)$$

Both Eq. (33) and Eq. (34) can be expressed as the following type:

$$\sum_{-M}^{N} G_{1n}A_n + \sum_{n=1}^{K} G_{2n}B_n = f_o$$
$$\sum_{-M}^{N} G_{3n}A_n + \sum_{n=1}^{K} G_{4n}B_n = g_o \quad (35)$$

where f_0 and g_0 are specified values at collocation point.

In general, the computed boundary values have errors at any collocation point ξ_m because the terms of stress function were truncated. The square of this error is

$$e_m^2 = \left[f_o - \sum_{-M}^{N} G_{1n}A_n - \sum_{n=1}^{K} G_{2n}B_n\right]_m^2 + \left[g_o - \sum_{-M}^{N} G_{3n}A_n - \sum_{n=1}^{K} G_{4n}B_n\right]_m^2 \quad (36)$$

The coefficients are determined by minimizing the squares of the errors at a specified number of points M_t on the boundary.

$$\frac{\partial \sum_{m=0}^{M_t} e_m^2}{\partial A_q} = 0 \quad (q = -M,\ -M+1,\ \cdots\cdots,\ N)$$
$$\frac{\partial \sum_{m=0}^{M_t} e_m^2}{\partial B_q} = 0 \quad (q = 1,\ 2,\ 3,\ 4,\ \cdots\cdots,\ K) \quad (37)$$

From the above equations, we obtain a set of $(N+M+K+1)$ linear algebraic equations for the unknown coefficients A_n and B_n.

$$\sum_{-M}^{N} \alpha_{nq}A_n + \sum_{n=1}^{K} \beta_{nq}B_n = \delta_q$$
$$\sum_{-M}^{N} \beta_{qn}A_n + \sum_{n=1}^{K} \gamma_{nq}B_n = \eta_q \quad (38)$$

where

$$\alpha_{nq} = \sum_{m=0}^{M_t}\left[G_{1n}G_{1q} + \sum_{n=1}^{K} G_{3n}G_{3q}\right]_m$$
$$\beta_{nq} = \sum_{m=0}^{M_t}\left[G_{2n}G_{1q} + \sum_{n=1}^{K} G_{4n}G_{3q}\right]_m$$
$$\gamma_{nq} = \sum_{m=0}^{M_t}\left[G_{2n}G_{2q} + \sum_{n=1}^{K} G_{4n}G_{4q}\right]_m$$
$$\delta_q = \sum_{m=0}^{M_t}\left[f_0 G_{1q} + \sum_{n=1}^{K} g_0 G_{3q}\right]_m$$
$$\eta_q = \sum_{m=0}^{M_t}\left[f_0 G_{2q} + \sum_{n=1}^{K} g_0 G_{4q}\right]_m \quad (39)$$

We obtained the final Eq. (38) to solve in this analysis. In applying the method, we must specify the collocation points on the boundaries at which the error equation is to be evaluated. Since stress symmetries were considered in Eq. (23), it is necessary to consider only a quarter of the plate. The collocation points on the circular hole boundary were specified by dividing the angular section into equal increments. This procedure was also applied to the external contour. In general, the total number of collocation points taken, M_t, is twice the total number of unknown coefficients in the stress function(Bowie and Freese, 1972).

5. STRESS INTENSITY FACTOR

The stress intensity factors may be evaluated directly from the stress functions $\phi_1(z_1)$ or $\phi_2(z_2)$. In the limit as z_j approaches the crack tip, say $z_0(=L)$, we can express the relation between the stress intensity factors and the stress function as follows(Sih and Liebowitz, 1978):

$$K_I + \frac{K_{II}}{s_2} = 2\sqrt{2\pi}\left[\frac{s_2 - s_1}{s_2}\right]\lim_{z_1 \to z_0}\sqrt{z_1 - z_0}\,\phi_1'(z_1) \quad (40)$$

Considering mapping function $z = \omega(\zeta)$ and employing Eqs. (12)~(15), we obtain

$$K_I + \frac{K_{II}}{s_2} = 2\sqrt{\pi/L}\left[\frac{s_2 - s_1}{s_2}\right]\phi_1'(1) \quad (41)$$

Substituting Eq. (23) into Eq. (36) and considering only the opening mode, the stress intensity factor can be expressed in terms of coefficients of stress function as follows:

$$K_I = 2\sqrt{\pi/L}\left[\frac{s_2 - s_1}{s_2}\right]\left[\sum_{-M}^{N}(2n+1)A_n + \sum_{n=1}^{K}\frac{(1-n)}{2^n}B_n\right] \quad (42)$$

Thus, we can evaluate the stress intensity factors if the coefficients of the stress functions are determined.

6. NUMERICAL RESULTS

We applied a modified mapping—collocation method to the analysis of cracks emanating from a circular hole in an orthotropic finite plate under uniform stress. To check the

Table 1 Correction factors (K_I/K_I°) obtained by truncating the terms of stress function
$\phi_1(\zeta) = \sum_{-M}^{N} A_n\zeta^{2n+1} + \sum_{1}^{K} B_n\zeta(\zeta^2+1)^{-n}$

M, N \ K	10	15	20	25	30
6	1.054	1.042	1.035	1.030	1.026
9	1.068	1.061	1.056	1.054	1.053
12	1.073	1.066	1.062	1.061	1.061
15	1.074	1.067	1.064	1.062	1.062
18	1.074	1.067	1.064	1.063	1.063
21	1.074	1.067	1.064	1.063	1.063
24	1.074	1.067	1.064	1.063	1.063

Laminate [0], $H/W=3$, $2R/W=0.2$, $2L/W=0.24$, $K_I^\circ = T\sqrt{\pi L}$

Table 2 Correction factors (K_I/K_I°) obtained by truncating the terms of stress function $\phi_1(\zeta)=\sum_{-M}^{N} A_n\zeta^{2n+1}+\sum_{1}^{K} B_n\zeta(\zeta^2+1)^{-n}$

M, N \ K	10	15	20	25
6	1.374	1.370	1.366	1.363
9	1.417	1.415	1.412	1.410
12	1.454	1.453	1.452	1.451
15	1.473	1.473	1.473	1.472
18	1.484	1.483	1.483	1.483
21	1.489	1.489	1.489	1.489
24	1.493	1.493	1.493	1.493
27	1.494	1.494	1.494	1.494

Laminate [0], $H/W=3$, $2R/W=0.2$, $2L/W=0.7$, $K_I^\circ=T\sqrt{\pi L}$

effectiveness of this procedure, we presented the various results for comparison with references. Then, the stress intensity factors were presented for several plate configurations of $[0_n/90_m]_s$ laminates.

Tables 1 and 2 show the correction factors for short and long cracks respectively, which were obtained by truncating the terms of stress function. It was appropriate to truncate at $M=21$, $N=21$, and $K=20$ for $2L/W<0.4$ and $M=24$, $N=24$, and $K=15$ for $2L/W\geq 0.4$. Material properties of E-glass/epoxy used in the current analysis are as followings:

$$E_1=53.74\ \text{GPa}(7.80\times 10^6\text{psi}),$$
$$E_2=17.91\ \text{GPa}\ (2.60\times 10^6\ \text{psi}),$$
$$G_{12}=8.96\ \text{GPa}(1.30\times 10^6\ \text{psi}),$$
$$\nu_{12}=0.25.$$

Figures 3, 4 and 5 compare the present results with those of references. Figure 3 shows the correction factors for cracks emanating from a circular hole in an isotropic finite plate under uniform stress. The present results almost coincide with Newman's results(Newman Jr., 1971), which were obtained by using the boundary collocation method. The isotropic solution was obtained by setting the complex parameters $s_1=1.0i$ and $s_2=0.995i$. Figure 4 shows the correction factors for cracks emanating from a circular hole in glass/epoxy unidirectional laminate under uniform stress. The present result is in good agreement with Wang's result(Wang and Yau, 1980), which was obtained by using the finite element method. In Fig. 4, Wang's result for glass/epoxy was replotted by digitizing their graph. Figure 5 shows the correction factors for central cracks in an isotropic finite plate. The present results were obtained by taking the radius of hole, R as small value. The present results almost coincide with Bowie's results(Bowie and Freese, 1972), which were obtained by using the modified mapping—collocation method

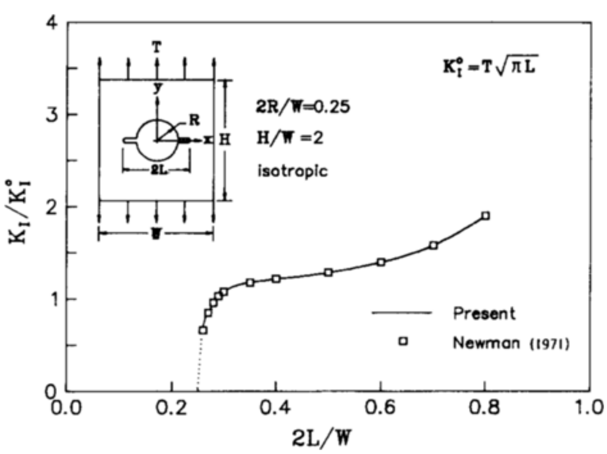

Fig. 3 Correction factors for cracks emanating from a circular hole in an isotropic finite plate under uniform stress

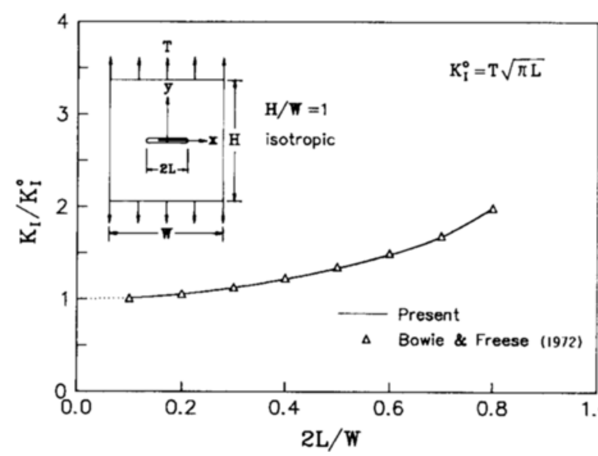

Fig. 5 Correction factors for a central crack in an isotropic finite plate under uniform stress

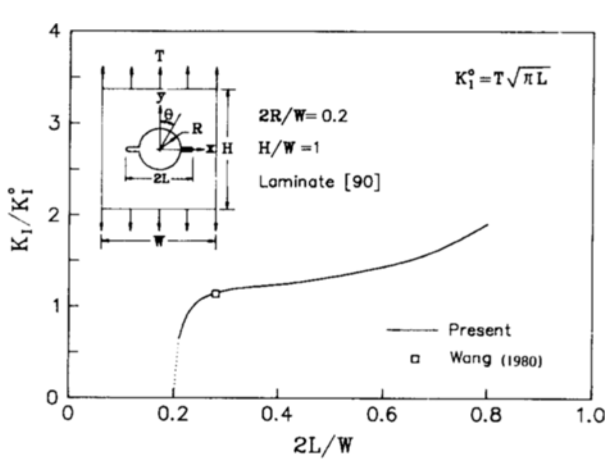

Fig. 4 Correction factors for cracks emanating from a circular hole in laminate [90] under uniform stress

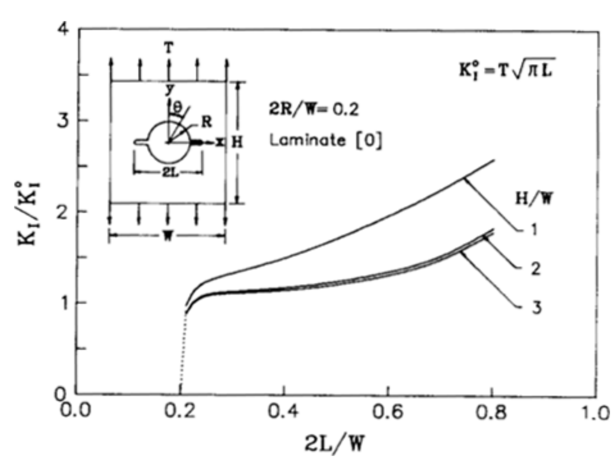

Fig. 6 Correction factors for cracks emanating from a circular hole in laminate [0] under uniform stress

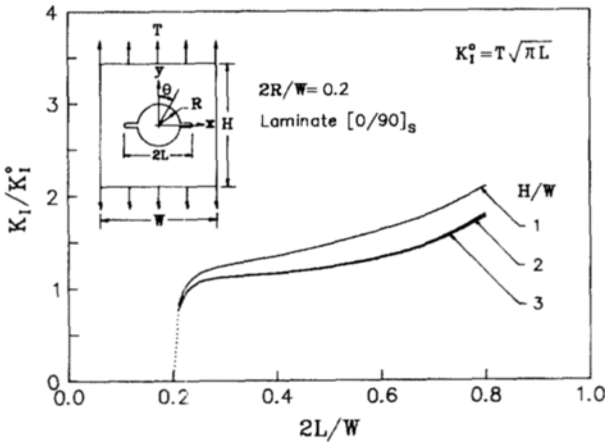

Fig. 7 Correction factors for cracks emanating from a circular hole in laminate $[0/90]_s$ under uniform stress

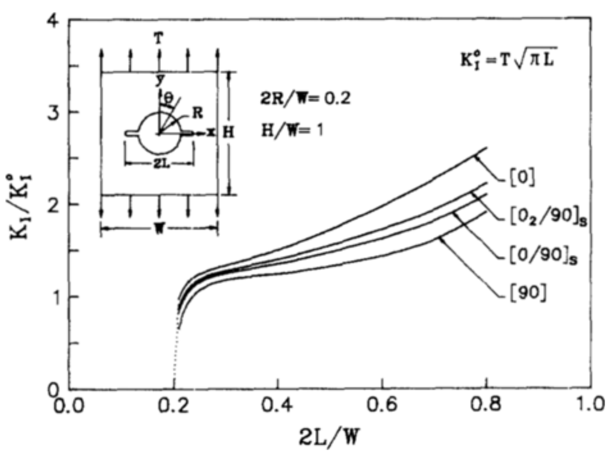

Fig. 8 Correction factors for cracks emanating from a circular hole in cross-ply laminate $[0_n/90_m]_s$ under uniform stress

and employing the simple stress function formulated for only a central crack. Observing Figs. 3, 4 and 5, we can see that the present results almost coincide with those of references.

Figures 6 and 7 show the correction factors for cracks emanating from a circular hole in laminate [0] and laminate $[0/90]_s$ under uniform stress. The stress intensity factors decrease as the aspect ratio, H/W increases. In Fig. 6 the correction factors have limits when the aspect ratio, H/W is about 3. In Fig. 7 the correction factors have limits when the aspect ratio, H/W is about 2.

Figure 8 shows the correction factors for cracks emanating from a circular hole in $[0_n/90_m]_s$ laminates with $H/W=1$ under uniform stress. In the range of entire crack length, the stress intensity factors for $[0_n/90_m]_s$ family exist between those for $\theta=0^0$ and $\theta=90^0$. The more the percentage of 0^0 plies increases, the larger the stress intensity factor becomes. Figure 9 shows the correction factors for cracks emanating from a circular hole on $[0_n/90_m]_s$ laminates with $H/W=2$ under uniform stress. In the range of small crack length, the stress intensity factors for $[0_n/90_m]_s$ family exist between those for $\theta=0°$ and $\theta=90°$ and the stress intensity factor for $\theta=0°$ is about 40 percent larger than that for $\theta=90°$. But, as the crack length increases, the stress intensity factors for $[0_n/90_m]_s$ family almost make no difference.

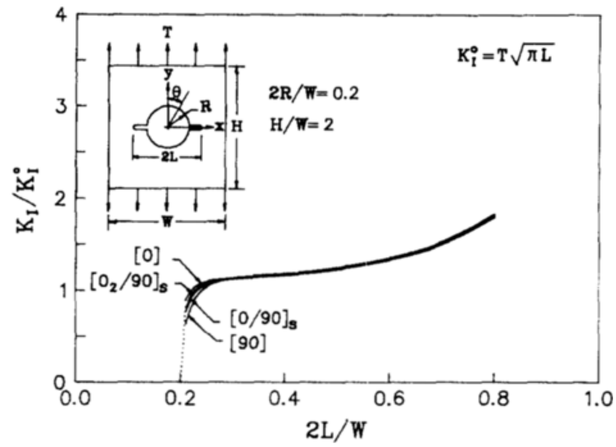

Fig. 9 Correction factors for cracks emanating from a circular hole in cross-ply laminate $[0_n/90_m]_s$ under uniform stress

7. CONCLUSION

A modified mapping-collocation method was applied to the analysis of cracks emanating from a circular hole in an orthotropic finite plate under uniform stress. Comparing the present results with those of references, we can see that this method is effectively applicable to analyzing such cracks.

As the aspect ratio increases, the stress intensity factors for cracks emanating from a circular hole in $[0_n/90_m]_s$ laminates under uniform stress decrease. In the range of entire crack length, the stress intensity factors for $[0_n/90_m]_s$ laminates with H/W=1 exist between those for $\theta=0°$ and $\theta=90°$. The more the percentage of 0° plies increases, the larger the stress intensity factor becomes. In the range of small crack length, the stress intensity factors for $[0_n/90_m]_s$ laminates with large aspect ratio exist between those for $\theta=0°$ and $\theta=90°$. But, as the crack length increases, the stress intensity factors for $[0_n/90_m]_s$ family almost make on difference.

The stress intensity factors for cracks emanating from a circular hole in an isotropic plate can be directly obtained by using the program developed here. The stress intensity factors for central cracks can be also obtained by using the same program.

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